THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics

MATH 2055 Tutorial 5 (Oct 19)

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1. Let a be a real number, and let f, g be differentiable on $(a-d, a+d)$ where d is a positive real number.
   If $g'(x) \neq 0$ and $g(x) \neq 0$ on $(a-d, a+d)$,
   $\lim\limits_{x \to a} f(x) = \lim\limits_{x \to a} g(x) = 0$ and $\lim\limits_{x \to a} \dfrac{f'(x)}{g'(x)} = \infty$,
   show that $\lim\limits_{x \to a} \frac{f(x)}{g(x)} = \infty$

2. Show that $\lim\limits_{x \to -1} \frac{x+5}{2x+3} = 4$

3. show that $\lim\limits_{x \to 0} \sin(1/x^2)$ does not exist.

4. Suppose f is a continuous function map $[0,1]$ to $[0,1]$.
   Show that there exists $x \in [0,1]$ such that $f(x) = x$.
   Is the statement still true if the interval is replaced by $[0,1] \cup [2,3]$?

5. Let f be continuous on the interval $[0,1]$ to $\mathbb{R}$ and such that $f(0) = f(1)$.
   Prove that there exists a point c in $[0, \frac{1}{2}]$ such that $f(c) = f(c + \frac{1}{2})$.